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The Role of Polysemioticity in the Effectiveness of Teaching Aspects of Mathematics: A Pilot Study Report

Rola polisemiotyczności w skuteczności nauczania aspektów matematyki. Raport z badania pilotażowego

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Abstract. Based on theories such as those of Daniel Ginsberg and Zenon Dyrszlag, this article examines the potential of polysemiotic teaching to improve the effectiveness of mathematics education and conceptual understanding, using basic concepts from metric space theory as an example. In a pilot study (N = 11), more polysemiotic methods (experimental group; n = 4) were compared with less polysemiotic methods (control group; n = 7). The experimental group achieved higher scores on the tests, suggesting that more polysemiotic teaching may enhance abstract conceptual understanding and problem-solving skills, offering strategies for improving the effectiveness of mathematics education.

Keywords: polysemiotic teaching; conceptual understanding; effectiveness of teaching; mathematics education; metric spaces

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Abstrakt. W artykule, odwołując się do teorii takich badaczy jak Daniel Ginsberg i Zenon Dyrszlag, przedstawiono, w jaki sposób nauczanie polisemiotyczne może zwiększać efektywność nauczania i pogłębiać rozumienie pojęć matematycznych. Zagadnienie to zilustrowano na przykładzie podstawowych pojęć z teorii przestrzeni metrycznych. W badaniu pilotażowym (N = 11) porównano metody bardziej polisemiotyczne (grupa eksperymentalna, n = 4) z metodami mniej polisemiotycznymi (grupa kontrolna, n = 7). Testy wykazały wyższe wyniki w grupie eksperymentalnej, co wskazuje, że bardziej polisemiotyczne nauczanie może pogłębiać abstrakcyjne rozumienie pojęciowe i zdolność rozwiązywania zadań, oferując strategie podnoszące efektywność edukacji matematycznej.

Słowa kluczowe: nauczanie polisemiotyczne; rozumienie pojęciowe; efektywność nauczania; edukacja matematyczna; przestrzenie metryczne

INTRODUCTION

Teaching abstract mathematical concepts requires methods that transcend traditional linguistic and symbolic instruction. While multimodality is recognized in education, i.e., engaging multiple senses, the specific role of polysemiotticity, such as the integrated use of diverse semiotic systems like gestures, visuals, and artifacts, remains underexplored in mathematics education, particularly with regard to advanced topics, such as metric spaces. This gap is important because mathematical thinking itself involves translating between representations and different semiotic systems, a process that teaching should explicitly facilitate.

This study proposes that intentionally polysemiotic teaching can significantly enhance conceptual understanding and thereby improve performance on tests requiring different levels of comprehension. This paper presents a pilot study in which more polysemiotic teaching methods were compared with less polysemiotic ones. The primary aim was to gather preliminary evidence on their effectiveness and assess the feasibility of the experimental design for a larger-scale investigation. The results, although based on a small sample, suggest that the use of polysemiotic methods not only improves test scores but also enhances subjective perceptions of understanding.

THEORETICAL FRAMEWORK: POLYSEMIOTICITY AND MEANING-MAKING IN MATHEMATICS

1. Definition of polysemioticity, semiotic resources, and semiotic systems

In the context of polysemiotic analysis, establishing clear definitions is crucial, especially since scholars often use inconsistent terminology for related concepts. The term "polysemioticity" refers to the simultaneous use of multiple semiotic systems in communication, e.g., spoken language, diagrams, or gestures.

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This definition was proposed by Jordan Zlatev and has been adopted by numerous researchers in the field. In instances where multiple semiotic systems are employed, communication becomes polysemiotic (Li, Zlatev 2022).

It is important to distinguish polysemioticity from multimodality. Polysemioticity involves the interplay of semiotic systems and resources (e.g., language, gesture, notation), while multimodality refers to the engagement of multiple sensory modalities (e.g., sight, touch, hearing, smell) to perceive a message (Stampoulidis 2020). Therefore, analyzing multimodality focuses on which sensory modalities are engaged, whereas analyzing polysemioticity focuses on the semiotic resources and systems involved. Semiotic resources are actions or materials with meaning potential, actualized in social contexts (Leeuwen 2005). In contrast, semiotic systems use these resources normatively and systematically, so their use and interpretation are less situational or idiosyncratic. Within systems, signs have specific affordances and interpretations (Stampoulidis 2019; Zlatev 2019). For example, colored blocks are a semiotic resource, and traffic lights constitute a semiotic system.

2. Meaning-making of mathematics

Mathematical language and notation are among the tools for representing and communicating knowledge about the world (Ginsberg 2015). For example, they help students articulate mathematical phenomena with precision and enable the development of appropriate reasoning strategies. This includes understanding not just how, but also why, a particular formula or symbol is used.

In the mathematics classroom, semiotic systems can operate simultaneously: speech and its prosody, written text, formal notation, visual imagery, gesture, body orientation, eye gaze, digital tools, and even the spatial organization of the classroom. All of these contribute to the process of meaning-making and are situated within an interactional context.

Thus, meaning-making in mathematics necessarily involves multiple semiotic systems. Although Daniel Ginsberg (ibidem) frames this in terms of multimodality, his insights can and should be extended to the concept of polysemioticity. While multimodality emphasizes different sensory modalities, polysemioticity focuses on semiotic systems and their interactions. Both concepts are deeply intertwined in educational practice and provide additional opportunities.

This is supported by research showing that abstract mathematical concepts are often understood through embodied metaphors and that gestures can bridge the gap between intuition and formalism, which is especially important in mathematics (Lakoff, Núñez 2000; Radford 2009). George Lakoff and Rafael E. Núñez (2000) argue that abstract mathematical concepts are grounded in embodied and

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metaphorical experience, which directly supports at least the use of gestures and physical artifacts in effective teaching. Although it remains necessary to explore how specific resources are used to construct meaning (Magnusson, Godhe 2019), it is reasonable to argue that meaning-making, particularly in mathematics, is both a multimodal and polysemiotic process.

Consider, e.g., the concept of distance, which is fundamental to the theory of metric spaces. Explaining this concept solely through natural language is extremely difficult, if not impossible. Visual representations are essential to conveying how the concept operates. However, without mathematical notation, the explanation loses mathematical precision.

Thus, meaning-making in mathematics necessarily involves multiple semiotic systems. This is why a polysemiotic approach is essential. Furthermore, research indicates that students employ diverse cognitive strategies (e.g., verbal, visual, algebraic), suggesting that teaching that engages multiple systems is more likely to support learners with varying preferences (Goodson-Espy 1998; Grandin 2022). This theoretical framework informs the design of the present pilot study, which compares a more polysemiotic method of teaching with a less polysemiotic one. A more polysemiotic method involves the use of a greater variety of semiotic systems and resources during a lesson. This distinction will be explained in more detail below.

The above considerations support the conclusion that teaching mathematics, especially the more abstract branches such as the theory of metric spaces – whose basic concepts are the subject of the study – is an inherently polysemiotic process. Purposefully enriching instruction with additional sign systems (gestures, visualizations, artifacts) should, therefore, facilitate deeper knowledge integration and its better transfer, forming the basis for the research hypothesis.

3. Levels of understanding and assessment in mathematics

The concept of levels of understanding, as used here, can be specified by the following classification. Four distinct levels of mathematical understanding can be identified (Dyrszlag 1974):

1. Definition level, which stands for a bare minimum, e.g., reproducing function definitions.
2. Local complication level. The student can create examples and non-examples while meeting some extra conditions.
3. Generalization level. The student can prove theorems concerning the concept because of the transition from concrete to general. For example, proving properties of some operations.

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4. Structural level. The student can see the structure of the concept and recognize analogous structures in different mathematical objects. It is the highest level of mathematical understanding. It is nearly impossible to achieve in school. For example, function–matrix analogies can then be observed.

When designing assessments, both the level of understanding targeted and the nature of the cognitive processes involved must be considered. Additionally, the taxonomy of learning outcomes includes habitual responses, more complex stereotypical operations, and creative or transfer-level thinking.

Effective assessment requires more than checking for memorized answers. Exercises should be novel enough to prevent automatic recall and instead stimulate students' reasoning. Students must be able to parse the logical and linguistic structure of questions and be familiar with the symbols and terms involved. Yes–no questions, unless paired with justification, are of limited diagnostic value (ibidem).

Several assessment methods correspond to different cognitive levels:

1. Definition-based exercises may ask students to: (a) state a definition in their own words; (b) select correct definitions among alternatives; (c) explain why a given object fails to satisfy a definition; or (d) recognize redundancies in a definition (e.g., one condition implied by another).
2. Example-based exercises may involve: (a) generating multiple, varied examples of a concept; (b) modifying non-examples into valid instances; and (c) choosing valid examples from a set of possibilities.
3. Counterexample-based exercises include: (a) finding objects that nearly meet a definition, but fail due to one condition; (b) adjusting examples to turn them into counterexamples; and (c) explaining what disqualifies a given object.
4. The reverse exercises test conceptual flexibility. They may ask students to: (a) translate verbal descriptions into equations (e.g., "Lucy bought five notebooks and a pencil case…"); (b) derive descriptions from symbolic representations; (c) draw a figure from a verbal description; or (d) interpret a given diagram or figure verbally.

4. Theory of metric spaces as an aspect of mathematics

The basic concepts used in the study – distance, metrics, and circle – are fundamental to the theory of metric spaces, a branch of mathematics that introduces rigorous ways of measuring distances between points in a set. A metric space is defined by a set equipped with a distance function (a metric) that satisfies three conditions: identity of indiscernibles (distance from a point to itself is zero), symmetry, and the triangle inequality. These seemingly simple axioms

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give rise to rich geometric and topological structures, including the concepts of open sets, continuity, and convergence (Jackowski 2013).

The theory of metric spaces occupies an important position between geometry and topology. Geometry differentiates shapes based on rigid properties. For example, geometry distinguishes between a circle and a square, considering both their interiors and boundaries. Topology, by contrast, treats these shapes as equivalent, focusing on properties that remain invariant under continuous deformation – stretching, twisting, and bending, but not tearing or gluing (Frej 2018). Metric spaces provide a bridge between these two perspectives: they retain the quantitative notion of distance from geometry, while also serving as the primary source of examples for topological concepts such as continuity and compactness.

Understanding metric spaces is inherently abstract and often requires students to think about spaces and transformations in highly conceptual terms. Unlike elementary geometry, where concepts like scaling or rotation are relatively intuitive, metric spaces introduce unfamiliar ways of measuring distance (e.g., the Manhattan metric or the railway metric) that challenge everyday intuitions. Effective pedagogical approaches must therefore leverage polysemiotic meaning-making – combining clear verbal explanations with visualizations, notation, and physical artifacts – to bridge the gap between formal definitions and intuitive understanding (Lakoff, Núñez 2000; Radford 2009).

For the purposes of this study, only the most basic concepts of metric spaces were used: distance, circles (defined as sets of points at a fixed distance from a center), and metrics (including Euclidean, Manhattan, and railway metrics). These concepts are sufficient to illustrate the potential role of polysemiotic teaching in fostering conceptual understanding.

TEACHING ASPECTS OF MATHEMATICS – STUDY WITH THE USE OF DIFFERENT SEMIOTIC SYSTEMS AND RESOURCES

1. Methodology

A pilot study was conducted to test the feasibility of the experimental procedure and to gather preliminary data on the effectiveness of polysemiotic methods in teaching metric concepts. In the study, the more polysemiotic method of teaching was compared to the less polysemiotic one. This was a pilot study designed to verify tools and procedures before a full-scale project.

The study focused on students participating in a lesson incorporating varying levels of polysemioticity. It addressed two research questions:

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1. Are teaching methods that use a greater variety of semiotic systems and resources associated with a higher number of correct responses compared to those using less polysemiotic approaches?
2. Is this effect attributable to a deeper understanding of the concepts, as measured by tasks requiring different levels of conceptual comprehension?

The study of these questions involved two distinct components: subjective evaluations of understanding and objective test results following the lesson.

The research problem was defined as follows: Does using more polysemiotic methods in teaching mathematics positively influence students' task performance? Positive influence was operationalized as an increase in correctly answered questions and higher levels of conceptual understanding. "More polysemiotic" refers to the use of a greater variety of semiotic systems and resources. The study focused on selected metric concepts: distance, circles, and metrics (see Table 1).

Table 1. Comparison between more and less polysemiotic methods

	Less polysemiotic methods	More polysemiotic methods
Natural language	yes	yes
Mathematical notation	yes	yes
Gestures	yes	yes
Depiction	limited (only to Euclidean metric and distance)	extended (to demonstrate methods of measuring distance and circles in all metrics)
Physical artifact (ribbon)	no	yes

Source: Author's own elaboration.

Based on the theoretical background, the following hypothesis was formulated: The use of more polysemiotic methods in teaching selected metric concepts (distance, circles, and metrics) positively affects students' ability to perform related exercises. Definitions of effectiveness, polysemiotic methods, and selected mathematical concepts were discussed in earlier sections.

Before the study, participants provided informed consent and completed a questionnaire covering: age, educational level (high school or university), mathematics experience and achievements (school-level mathematics and final exam results), and familiarity with basic and metric mathematical concepts.

The study was conducted in accordance with ethical standards for educational research.

The study participants were recent high school graduates and university students. They were studying mathematics at different levels, from basic to advanced, depending on the individual, but none of them had prior experience in

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the theory of metric spaces. For this study, they participated in a lesson dealing with metric concepts, such as metrics, distance, and ways of measuring distance.

The participants were randomly assigned to one of two groups: a control group (n = 7; mean age = 21.4; SD = 3.309) and an experimental group (n = 4; mean age = 22.25; SD = 4.193).

Both groups participated in an online lesson via Google Meet, covering metric concepts (distance, metric, circle). During the lesson for the control group, the following tools were used: language system, gestural resources, depiction (excluding certain aspects, particularly circles, when different metrics were employed), and mathematical notation.

Firstly, the common understanding of distance and distances in the Euclidean metric was discussed. Also, examples of counting distances between points in the Euclidean metric were presented.

Secondly, the definition of a metric and the three conditions for a function be a metric were introduced: the distance from a point to itself is zero, symmetry (the distance from x to y is always equal to the distance from y to x), and the triangle inequality.

Thirdly, examples of non-Euclidean metrics, that is, the "Manhattan metric" (see Figure 1) and the "railway metric" (see Figure 2), were presented, along with examples showing how the distance between points is measured in the case of these metrics.

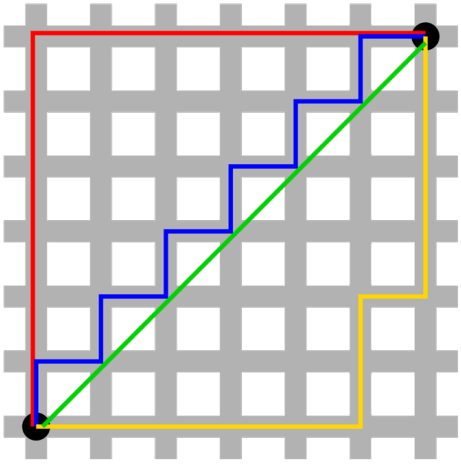


Figure 1. Manhattan metrics

Source: https://upload.wikimedia.org/wikipedia/commons/thumb/0/08/Manhattan_distance.svg/3840px-Manhattan_distance.svg.png?utm_source=pl.wikipedia.org&utm_campaign=index&utm_content=thumbnail

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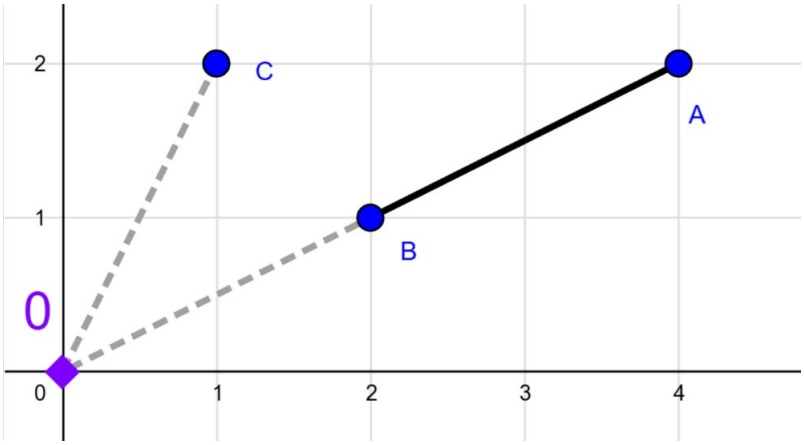


Figure 2. Railway metric

Source: https://www.geogebra.org/m/jnSwY8ca

Lastly, the definition of a circle was introduced, and how the shapes of different circles are determined in the case of each of the metrics was shown. Circles were described in the form of inequalities and written in mathematical notation. The meaning of each inequality was explained (e.g., a circle with a given center and radius, a line segment between two points), but without visual aids.

During the lesson with the experimental group, the same semiotic systems and resources were used as in the control group (natural language, gestural resources, and mathematical notation), with two additional components: extended visual representation (especially visualizations of circles in different metrics) and physical materials, the key physical artifact being a ribbon. The teacher used the ribbon to demonstrate how distance is measured in Euclidean space (along a straight line) and in the Manhattan metric (by "bending" it at right angles to simulate movement along a grid), thereby visually illustrating the differences in how distance is defined across metrics.

2. Measurement and data analysis

Right after the lesson, all participants completed an assessment test and a feedback questionnaire. The test consisted of eight exercises designed to measure understanding across three levels based on Zenon Dyrszlag's (1974) framework: definition (Exercises 1–4), local complication (Exercises 5, 6, and 8), and generalization (Exercise 7).

The test tasks were scored according to predefined criteria. For definition-based Exercises (1–3), each correct answer received 2 points; partially correct

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or incomplete answers (e.g., simplified definitions lacking full precision) received 1 point. Exercise 4 involved calculations. A total of 12 points were available, divided as follows: 3 points for correctly applying each of the three metrics (Euclidean, Manhattan, railway) to the given problem, plus 1 point for the accuracy of the calculations per metric. Thus, for each metric, a maximum of 4 points could be earned (3 for correct metric use + 1 for correct computation). For Exercise 5, participants were asked to identify sets of points belonging to a given circle (3 points in total, 1 point for each correct set) and to draw circles in all three metrics (9 points in total, 3 points for each of the three circles). In Exercise 6, participants had to recognize the metric from a visual representation of a circle. Each correct identification was worth 1 point, giving a maximum of 5 points. Exercise 7 required participants to determine whether a given formula defines a metric and to justify their answer. A maximum of 2 points was awarded: 1 point for correctly identifying whether the formula qualifies as a metric and for partially verifying the axioms, and 2 points for a fully correct solution with complete justification. Exercise 8 required generating non-examples of a circle in a given metric. Each valid non-example received 1 point.

All statistical analyses were performed using IBM SPSS Statistics. Independent samples *t*-tests were used to compare group means on the total test score and individual exercise scores. Levene's test was used to assess the homogeneity of variance. The alpha level was set at 0.05 for all statistical tests.

After completing the lesson and test, each participant filled out a feedback form. It contained two main parts:

1. Usefulness of resources. Participants rated the subjective level of usefulness of each resource individually (physical materials, Polish language, gestures, visualizations, and mathematical notation) on a scale from 1 to 10. Importantly, even students in the control group were asked to assess the usefulness of resources they did not encounter, to compare the perceived potential value across both groups. The term "semiotic system" was intentionally avoided in the questions.
2. Self-assessment of understanding. Participants rated their subjective sense of understanding for each concept taught during the lesson – distance, methods of measuring distance, Euclidean metric, Manhattan metric, railway metric, and the concept of a circle – again using a 1–10 scale.

This feedback aimed to explore possible correlations between participants' perception of their understanding and their actual performance on the test. Such correlations may provide additional insight into how different semiotic resources support learning.

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3. Results

3.1. Quantitative assessment

An independent-samples *t*-test revealed a statistically significant difference in total test scores between the groups. On a possible score range of 0–40 points, the experimental group achieved a mean of $M = 29.38$ ($SD = 5.5$), reaching 73.5% of the maximum score. The control group scored $M = 16.86$ ($SD = 5.89$), which corresponds to 42.2% of the maximum, resulting in a difference of 31 percentage points between the groups, $t(9) = 3.54$, $p = 0.003$ (see Tables 2 and 3). Cohen's d was calculated to assess the magnitude of the difference between groups. The analysis revealed a very large effect, Cohen's $d = 2.22$. Given the small sample sizes (particularly $n = 4$), Hedges' g correction for bias was applied, yielding $g = 1.92$, which still exceeds the conventional threshold for a large effect (>0.8). The 95% confidence interval for Cohen's d ranged from 0.68 to 3.76. While the effect remains very large, the wide confidence interval reflects substantial uncertainty due to the small samples, suggesting that the true population effect could be moderate or extreme.

Table 2. Averages of results

	Group	*N*	Average	Standard deviation
Final results	more polysemiotic methods	4	29.375	5.893
	less polysemiotic methods	7	16.857	5.513

Source: Author's own elaboration.

Table 3. Significance of differences between results in both groups

Test for independent samples											
		Levene's test of homogeneity of variances		Paired *t*-test for equality of averages							
						significance		difference in averages	standard error of difference	95% confidence interval for	
		F	significance	*t*	*df*	single-sided *P*	two-sided *P*			lower limit	upper limit
Final results	equality of variances was assumed	0.005	0.945	3.539	9	0.003	0.006	12.5179	3.5367	4.5174	20.5183
	equality of variances was not assumed	–	–	–	3.4696	0.007	0.013	12.5179	3.6089	3.6874	21.3484

Source: Author's own elaboration.

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Analysis at the exercise level showed that significant differences ($p < 0.05$) favoring the experimental group were found on exercises requiring higher-order application of concepts: specifically, Exercise 4 (calculation), Exercise 6 (identifying a metric from a visual representation of a circle), and Exercise 8 (generating a non-example). No significant differences were found on exercises assessing rote definitional knowledge (Exercises 1 and 3). The detailed results are presented in Table 4.

Table 4. Differences between groups' mean in exercises

Exercise	Level of understanding	Control group mean (*SD*)	Experimental group mean (*SD*)	*p*-value (single-sided)
1	definition	1.714 (0.488)	1.875 (0.25)	0.28
2	definition	1.286 (0.9512)	2.0 (0.0)	0.047
3	definition	1.5 (0.6455)	1.75 (0.2887)	0.201
4	definition	6.429 (2.8785)	9.625 (1.493)	0.036
5	local complication	3.929 (1.9457)	6.75 (4.3493)	0.82
6	local complication	1.0 (1.29)	4.25 (1.5)	0.002
7	generalization	0.857 (0.378)	1.125 (0.8539)	0.241
8	local complication	0.143 (0.378)	2.0 (1.4142)	0.038
Total		16.86 (2.0837)	29.38 (2.94)	0.003

Source: Author's own elaboration.

Figure 3 shows the distribution of test results across the two groups. While the overall difference between the groups is visible, there are also edge cases where one group gets a result more in the range of another.

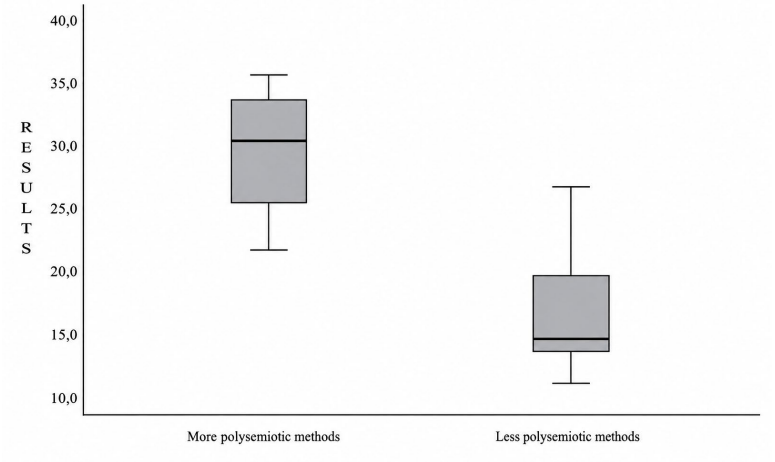


Figure 3. Distribution of results in groups

Source: Author's own elaboration.

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As presented in Figures 4 and 5, answers to Exercise 4 were similar for both groups (the first level of understanding was needed there). For Exercise 8 (requiring the second level of understanding), answers in the control group typically consisted of responses like "I don't know" (see Figure 6), whereas the experimental group usually provided substantive answers (see Figure 7). The contrast between the control group's "I don't know" responses and the experimental group's substantive answers (e.g., providing a specific non-example) suggests that polysemiotic teaching may have helped students reach the local complication level (Dyrszlag's level 2), whereas the less polysemiotic group often remained at the definition level.

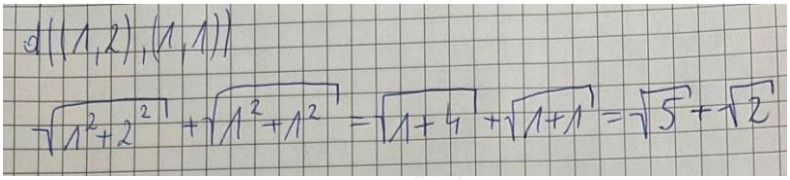


Figure 4. Example of answer (group 1, Exercise 4)

Source: students' answers.

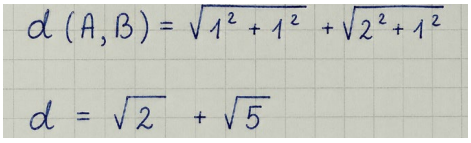


Figure 5. Example of answer (group 2, Exercise 4)

Source: students' answers.

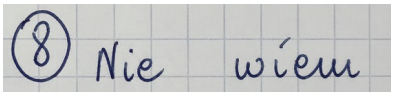


Figure 6. Example of answer (group 1, Exercise 8)

Source: students' answers.

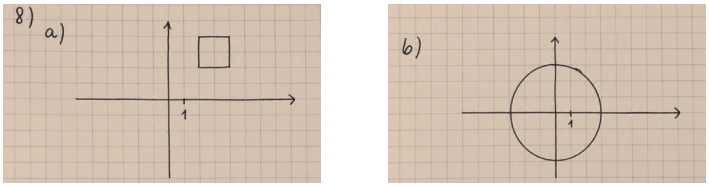


Figure 7. Example of answer (group 2, Exercise 8)

Source: students' answers.

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When examining each exercise separately, differences were observed between the group means for all exercises, with the group taught using more polysemiotic methods consistently achieving higher scores. The differences were as follows: Exercise 1 – 0.1607 points (out of 2 possible), Exercise 2 – 0.7143 points (out of 2 possible), Exercise 3 – 0.25 points (out of 2 possible), Exercise 4 – 3.1964 points (out of 12 possible), Exercise 5 – 2.8214 points (out of 12 possible), Exercise 6 – 3.25 points (out of 5 possible), Exercise 7 – 0.2679 points (out of 2 possible), Exercise 8 – 1.8571 points (out of 3 possible). However, this difference is statistically significant only for Exercises 2, 4, 6, and 8 (see Table 4).

Another important factor was the participants' perception of the usefulness of different semiotic systems and resources for the process of understanding. Differences in the mean usefulness ratings were observed for all the systems and resources considered. Again, in almost every case, the mean rating was higher in the group that used more polysemiotic methods (see Table 5).

Table 5. Significance of differences between personal feelings about the usefulness of different semiotic systems and resources

Semiotic system or resource	Significance – single-sided *P*	Statistical significance
Physical resources	<0.001	yes
Polish language	0.061	no
Gestures	<0.001	yes
Depiction	0.006	yes
Notation	0.03	yes

Source: Author's own elaboration.

There was also a difference between the means when the level of mathematics at school was taken into account, but the difference was not statistically significant ($p = 0.071$). However, in future research, it will be essential to control for this factor.

In the group taught using fewer polysemiotic methods, some students did not attempt to answer the question. Participants in this group also claimed in the place of response in the test or during the lesson: "I do not understand", "I have no idea", and "It is hard to tell". In Exercise 6, one participant wrote, "It's hard to tell because, other than the formulas, I still don't know what these metrics are". In contrast, participants in the experimental group made comments such as, "It requires thought and familiarity, but it makes sense".

Thus, the results may indicate that more polysemiotic teaching methods are associated with higher final scores, which may also indicate a higher level of understanding of the mathematical concepts taught.

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Looking in more detail:

1. More polysemiotic methods were associated with higher final test scores. For every individual exercise, the group taught using more polysemiotic methods achieved higher scores; however, the difference was not statistically significant for every exercise.
2. The influence of using more polysemiotic methods was noticeable in Exercises 6 and 8 (6 – state in which metric the given circle is drawn, 8 – give an example of an object that is not the image of a circle in the given metric), where a second level of understanding was needed, so a local complication level. There, understanding what an example of the concept is and what is not an example was crucial.
3. More polysemiotic methods are associated with a higher perceived usefulness of different semiotic systems, not only in the case of physical resources and representations, whose presence or level of use was the object of manipulation during the study, but also in the case of gestures (which could be related to object manipulation during the experimental group lesson) and notation. This could indicate a generally higher level of personal sense of understanding, where all used semiotic systems and resources seem helpful in putting together a sense of understanding. Self-reported understanding is defined here as internal awareness of the meaning of a concept and the feeling that one would be able to make a statement about this concept, explain what it is, what properties it has, and what relationships exist between it and other concepts.

In Exercise 7 (justifying whether a given formula is a metric or not), the third level of understanding would be required. The results suggest that the participants lacked a clear strategy for justifying their answer choice.

Exercises 1–4 required the first (definition) level of understanding. In Exercises 1 and 3, there were no significant differences. Moreover, differences observed in Exercise 4 can be attributed to calculation errors and oversights. However, a higher level of understanding could also contribute to greater accuracy, owing to a better understanding of what the notation used represents. Similarly, in Exercise 2, knowing how metrics work in practice could help participants define them.

The results may have been influenced by the asymmetry in group size (7 vs. 4). A smaller experimental group increases the risk of Type II error (failure to detect significant differences), suggesting that in a full-scale study, the effect of using more polysemiotic methods could be even more pronounced.

However, the results should be interpreted with caution due to the following limitations: (1) the absence of a pretest means that we cannot determine whether

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the groups were equivalent at baseline; (2) the small and unequal sample sizes (4 vs. 7) increase the risk of Type II error and reduce statistical power; (3) all sessions were taught by a single teacher, leaving the teacher effect uncontrolled; (4) the intervention was delivered online, which limited access to physical materials; and (5) the absence of blind scoring (i.e., the same teacher may have assessed the responses) introduces the potential for detection bias.

DISCUSSION OF FINDINGS

The preliminary results are promising and suggest that the use of more polysemiotic methods may have a positive impact on teaching outcomes. This is consistent with the concept of a "semiotic bundle" (Arzarello, Paola, Robutti, Sabena 2008). Crucially, the effect was most pronounced in exercises requiring higher-order understanding (e.g., translation between representations), supporting the theory that polysemioticity may facilitate deeper conceptual integration. Despite randomization, the small sample sizes may have introduced bias into these results.

Although the results are promising and show a very large effect favoring the experimental condition, they must be interpreted with extreme caution. Due to the very small and uneven sample size (N = 11, 4 vs. 7), no causal conclusions can be drawn; the results serve only to generate hypotheses for a fully powered study. A significant limitation was the lack of a standardized pre-test to establish group equivalence in terms of prior mathematical knowledge and skills. This is important because the level of school mathematics (basic vs. advanced) could potentially influence the outcomes. These factors are inherent to the pilot nature of this work, but must be addressed in future research. Replication on larger, more balanced samples is necessary before any generalizable claims can be made.

IMPLICATIONS FOR PRACTICE

Despite its limitations, this study offers valuable preliminary insights. For teachers, it suggests that abstract concepts like metrics require polysemiotic support. Teachers should combine mathematical language with gestures, visualizations, and physical artifacts. For example, teachers can use a ribbon to visualize the "Manhattan" metric to show why the distance is computed by $|x_2-x_1| + |y_2-y_1|$.

Especially when exercises assess higher-level understanding (e.g., generating examples), they should be taught polysemiotically.

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CONCLUSIONS

This pilot study provides initial, promising evidence that enriching instruction with diverse semiotic systems can enhance understanding of advanced mathematical concepts such as those encountered in the theory of metric spaces. While the small sample necessitates caution, the observed effect indicates this may be a highly fruitful area for further research. A fully powered, randomized controlled trial is the essential next step to confirm these preliminary findings.

One possible explanation for this effect is rooted in the way the human brain processes information: more complex stimuli tend to activate additional regions of the brain (Gazzaniga, Mangun 2014), and the holistic character of certain mathematical concepts makes them difficult to capture through a single semiotic system or modality alone.

At the same time, it is important to remember that teaching is a complex process, and many variables can influence its outcomes. Not only are there limitations stemming from the pilot nature of this study, but also factors such as the student's attitude, mental and physical condition, environmental context, prior experience, and many others can play a significant role. As a result, when investigating teaching effectiveness, these factors should be carefully considered, controlled, or measured to the extent possible.

ANNEX

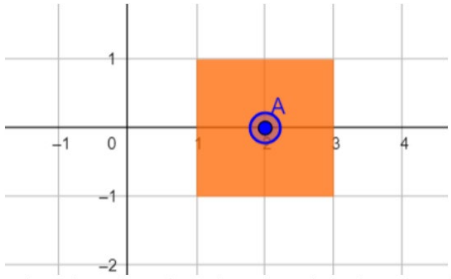
Test of knowledge

1. Provide a definition of distance.
2. Provide a definition of metric.
3. Provide a definition of circle.
4. Compute the distance between points in Euclidean metric, "railway" metric, and "Manhattan" metric:
   a. A = (1,1), B = (2,1)
   b. A = (−1,1), B = (−3,3)
   c. A = (−3,2), B = (4,−1)
5. Compute the sets of points included in given circle and draw this circle (in all of known metrics):
   a. B ((0,0), 2)
   b. B ((−1,−1), 3)
   c. B ((3,0), 1)

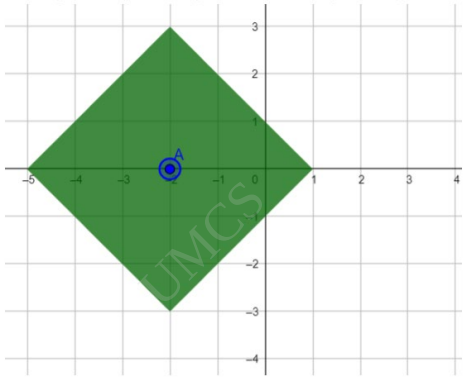
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6. Identify which of the three known metrics, if any, was used to draw the circle:

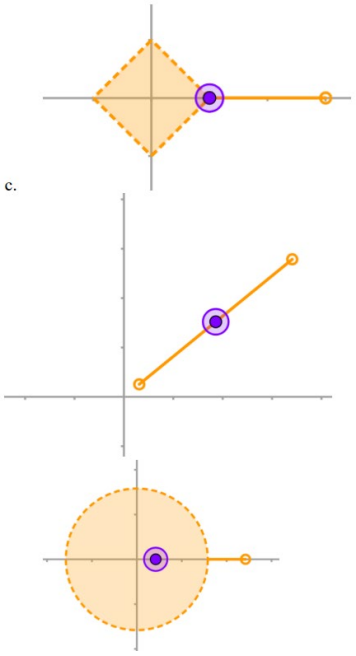
a.



b.



c.



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7. Justify why it is or it is not a metric:

$$d((x_1, y_1), (x_2,y_2)) = x_1 + y_1 + x_2 + y_2$$

8. Give an example of an object that is not a circle with respect to the given metric:

a. Euclidean
b. railway
c. Manhattan

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